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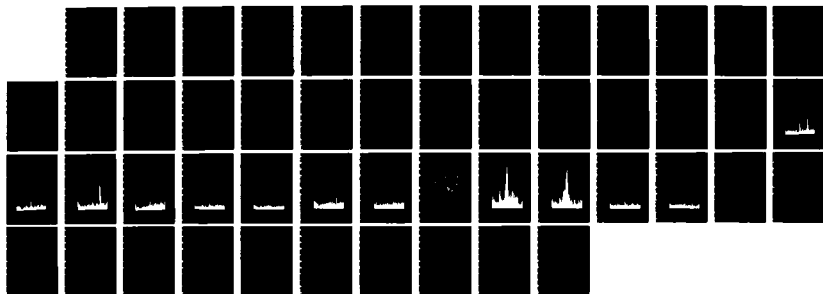
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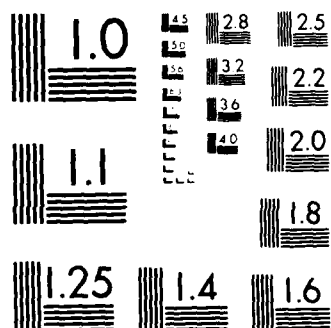
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# The Construction of Low Noise Optical Correlation Filters and Their Application to Target Identification Problems

**AD-A170 629**

Robert R. Kallman

NORTH TEXAS STATE UNIVERSITY  
DENTON, TEXAS 76203

JULY 1986

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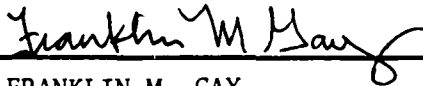
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## PREFACE

This document describes the results of an effort at the Department of Mathematics, North Texas State University, Denton, Texas 76203 under U.S. Air Force Grant F30602-81-C-0206. This effort originated at the Electro-Optical Terminal Guidance Branch, Advanced Sensor Division, U.S. Air Force Armament Laboratory, Eglin AFB, Florida, and was administered through RADC and the University of Dayton Postdoctoral Program.

The work reported herein was performed during the period 1 June 1985 to 31 December 1985 by the author, Dr. Robert R. Kallman, Professor of Mathematics. The report was released by the author in January 1986.

The author was introduced to this topic of research during the summer of 1984 while a SCEEE Postdoctoral Fellow at the U. S. Air Force Armament Laboratory, Eglin AFB, Florida. He gratefully acknowledges the continued support of the Armament Laboratory since that time.

The author would like to thank the Air Force Systems Command and the Air Force Office of Scientific Research for the opportunity to spend a very worthwhile and interesting ten weeks at the Armament Laboratory during the summer of 1984. The author would like to acknowledge in particular the Electro-Optical Terminal Guidance Branch and the Image Processing Laboratory for their hospitality and excellent working conditions. Special thanks go to Captain James Riggins and Steve Butler for introducing the author to the topic of optical correlation filters and patiently explaining its many aspects to him.

Continued thanks go to Peter Gianino of RADC, Hanscom AFB, and to Steve Butler, Dennis Goldstein, and Rick Wehling of the E-0 Terminal Guidance Branch and the SDI Branch of the Armament Laboratory for their patient support of the author and interest in his work over the past year and a half.



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## SECTION I

### INTRODUCTION

The work reported on in this document is the result of a mathematical approach to an engineering problem. The work is described in large part in mathematical language. The purpose of this section is to describe the general issues involved in the design of optical correlation filters. This will motivate the approach to the engineering problem being attacked.

A large body of work has been dedicated to shape-recognizing or object identification. One particular approach to this problem is optical correlation, based on the spatial frequency content of several views of the object. This is usually attempted in a holographic optical filter system which contains sufficient information about the object so that a suitably transformed image of the object can be processed by the filter. If the object sought is in the image, a large correlation peak should appear in the filter output.

The class of correlation filters invented by Caulfield and Maloney, Reference 1, and extrapolated by Caulfield and Weinberg, Reference 2, Hester and Casasent, Reference 3, and Leger and Lee, Reference 4, termed synthetic discriminant functions (SDF's), hold promise. Very roughly speaking, these filters attempt to pack a great deal of information into one device by extracting from several images of a target (the training set) that information which is unique about each of them and combine this information in a clever way to manufacture one filter which recognizes each image in the training set. However, these filters reportedly have been plagued with a low signal-to-noise

ratio (SNR - a precise mathematical definition is given in the next section); i.e., these filters have no problem correlating very well with true targets, but very often give high (even major) correlations with false targets. In fact, computer simulations run by the author on realistic data show that the standard recipe for constructing SDF's can give filters with an SNR of less than 1.0, even against a training set of images, in a zero background, which are edge-enhanced and energy-normalized. These low SNR's have been independently noted by Riggins and Butler, Reference 5. These facts make the standard recipe for constructing SDF's less than optimal.

This document outlines a general method which is a major variant of the standard recipe for manufacturing SDF's and seems to solve the SNR problem. In fact, when applied to the above mentioned data set, this new recipe leads to optical correlation filters which have an SNR over 7.37 against their training sets. Correlation filters with such SNR's might be useful for applications. The construction of these new filters is the result of the formulation of an engineering problem as a very complex mathematical minimax question. It is somewhat amusing that this mathematical minimax question can be interpreted as a very involved problem in plane geometry. This interpretation also makes intuitively clear why the new method might yield good results. The mathematical minimax question was solved for a fairly realistic test data set by elaborate computer programs which took a great deal of disk space and hundreds of hours of CPU time to run on a VAX 11-780/VMS 4.2. The long running time needed for the solution of the minimax question should not be a cause for alarm. The author has preliminary ideas which should cut the CPU time for the solution of the minimax question down to no

more than one-third its present length, even using the present computer configuration available to him. The time needed for the solution of the minimax question should be reduced by two orders of magnitude or more on a better configured computer system (even a VAX) with more physical memory and disk space available for the computation, using smaller (and more realistic) imagery in the training sets and even more advanced numerical techniques. The computation time should be further reduced by still another one or two orders of magnitude on a really advanced machine.

The design of a low noise (or high SNR) correlation filter is not a straightforward process. Extreme care should be taken to insure, to whatever extent possible, that the filter does not give strong positive signals to false targets. This is a rather complicated task since one does not know a priori all possible false targets - and even if one did know them and there were a large number of them, just how would one take all of the information into account in the design of the filter? The guiding philosophy behind the work in this document is that, in the absence of any a priori knowledge of the nature of false targets, one should design the filter to have a correlation output plane which looks as much as possible like a delta function centered over the target when the target is centered in the filter input plane. Furthermore, in the design process care should be taken to identify all possible independent parameters involved and optimize the design by varying these parameters.

If one adopts the design philosophy espoused in the previous paragraph, then one perhaps will design correlation filters which are somewhat sensitive to perturbations in the training set - i.e., the filter perhaps will give

major correlations only to images which are quite close to those used in its training set. This seems to be a necessary and worthwhile trade-off if one wants to have filters with a high SNR. This will be an especially worthwhile trade-off if low noise SDF filters can be effectively implemented in a high speed reusable spatial light modulator, Reference 6. Then one will have a device capable of displaying several hundred SDF correlation filters per second. If each of these displayed SDF filters can be manufactured from 10 to 100 training images and still have a high SNR one will have a device which will be able to search for  $10^3$  to  $10^4$  images per second with a low probability of committing an error. This perhaps should be enough for practical applications, especially if the spatial light modulator can be designed to permit minor on-the-spot programming to emphasize those SDF filters relevant to current weather and/or battle conditions.

## SECTION II

### THE MATHEMATICAL APPROACH TO THE PROBLEM

SDF's are usually manufactured by the following process. One starts with two-dimensional images  $f_1, \dots, f_n, \dots, f_m$ , often called the training set. The analysis of this section is entirely general and does not depend on any particular properties of the  $f_i$ 's - they could be ink spots, for example. It might be intuitively useful to view these  $f_i$ 's as either infrared images which are inputs to some filtering device or as square matrices, all of the same size, whose entries represent intensity levels. It is useful to view images as (in general real valued, but sometimes complex valued) functions on the plane, or, if they are large matrices, to string out the rows of the matrix and think of them as extremely large vectors in a very high dimensional complex inner product space. Here one should think of  $f_1, \dots, f_n$  as images of objects one is seeking and  $f_{n+1}, \dots, f_m$  as images of objects one is not seeking and definitely does not want to confuse with  $f_1, \dots, f_n$ .  $n$  may well be equal to  $m$ . An SDF  $h$  is also an image, artificially constructed and in general complex, in the author's formulation, implemented in a holographic optical correlation filter, which has certain properties. At the very least,  $h$  must satisfy the following condition: given positive numbers  $\lambda_1, \dots, \lambda_n$  a priori (which are not necessarily distinct, and in fact may all be equal to the same number, 1 say), one wants the intensities  $|\langle h, f_i \rangle|^2$  to satisfy

$$|\langle h, f_i \rangle|^2 = \lambda_i^2 \quad (1 \leq i \leq n). \quad (1)$$

Here, and in general, if  $f$  and  $g$  are two complex images,  $\langle f, g \rangle$  denotes the

complex inner product between the two images, viewed as complex valued functions on the plane or as large complex vectors, and the length or norm of  $f$  is given by  $||f|| = \langle f, f \rangle^{1/2}$ .  $||f||^2 = \langle f, f \rangle$  is the total energy in the image  $f$ . The condition (1) merely demands that if the image  $f_i$  is centered in the input plane, then the center pixel of the output correlation plane must have intensity  $\lambda_i^2$ .

Before doing anything else, let us find all possible solutions to the equations (1). If further constraints or requirements are later placed on  $h$ , they will be on the general solution of the equations (1). One must have

$$\langle h, f_i \rangle = z_i \lambda_i \quad (1 \leq i \leq n) \quad (2)$$

for some choice of complex numbers  $z_i$  of modulus 1. This gives us  $n$  simultaneous linear equations which  $h$  must satisfy. Look for a particular solution  $h_0$  of these equations of the form

$$h_0 = a_1 f_1 + \dots + a_n f_n, \quad (3)$$

for some choice of complex numbers  $a_1, \dots, a_n$ . To determine the  $a_i$ 's, plug equation (3) into the equations (2) and use  $\langle f_i, f_j \rangle = \langle f_j, f_i \rangle$ , since the  $f_k$ 's are real vectors, to get

$$(\langle f_i, f_j \rangle)(a_j) = (z_i \lambda_i). \quad (4)$$

If the  $n$  by  $n$  matrix  $(\langle f_i, f_j \rangle)$  is nonsingular (which is true if and only if the vectors  $f_1, \dots, f_n$  are linearly independent, as is almost surely the case in practice), then the equations (4) uniquely determine the  $a_j$ 's and thus the  $h_0$  which satisfies the equations (2) and equation (3). Let  $h$  be any other solution to the equations (2). A trivial computation shows that  $h' = h - h_0$  must be orthogonal to each  $f_i$  ( $1 \leq i \leq n$ ).

Thus, one may obtain every solution  $h$  of the equations (1) as follows.



Choose complex numbers  $z_1, \dots, z_n$  of modulus 1, determine  $h_0$ , as in equation (3), which solves the equations (2), by solving the simultaneous equations (4), choose  $h'$  orthogonal to each  $f_i$  ( $1 \leq i \leq n$ ), and let  $h = h_0 + h'$ . Every SDF  $h$  must be of this general form independent of any additional constraint or requirement which is put on it. This general setup of the SDF problem is the author's own variant of the ideas of Caulfield and Maloney, Reference 1, Caulfield and Weinberg, Reference 2, Hester and Casasent, Reference 3, and Leger and Lee, Reference 4. In particular, the idea of choosing  $h'$  orthogonal to the  $f_i$ 's and the  $z_i$ 's to be general complex numbers of modulus 1 rather than just identically 1 (the usual recipe) is the author's own idea and does not seem to have occurred to anyone else. The importance of this idea will become clearer later on.

Some notation is needed to describe other requirements on  $h$  and the main problem of this document. For any  $x$  in the plane and any image  $g$ , let  $g_x(y) = g(y - x)$  for any  $y$  in the plane. If one initially thinks of  $g$  as being centered over the origin, then think of  $g_x$  as being  $g$  translated so as to be centered over  $x$ . Ideally, to guarantee a high SNR, one wants that the measured optical intensities  $|\langle h_x, f_i \rangle|^2$  all be maximal for  $x$  the origin (note that  $h_{(0,0)} = h$ ) and that  $|\langle h_x, f_i \rangle|^2$  ( $1 \leq i \leq m$ ) be as small as possible for all  $x$ 's outside of some a priori chosen box (or any other region)  $B_i$  about the origin. Here  $B_i$  is empty for  $n+1 \leq i \leq m$ . The term SNR is now defined precisely, but somewhat arbitrarily, as follows. The SNR of  $h$ , a number intrinsically associated with  $h$ , is defined to be

$$\text{SNR}(h) = \min_{1 \leq i \leq m} \text{SNR}_i, \quad (5)$$

where  $\text{SNR}_i$ , the SNR of  $h$  against  $f_i$ , is defined by

$$\text{SNR}_i = \max_{x \in B_i} |\langle h_x, f_i \rangle|^2 / \max_{x \in B_i} |\langle h_x, f_i \rangle|^2 \quad (6a)$$

for  $1 \leq i \leq n$  and

$$\text{SNR}_i = \min_{1 \leq j \leq n} \lambda_j^2 / \max_x |\langle h_x, f_i \rangle|^2 \quad (6b)$$

for  $n+1 \leq i \leq m$ . Thus, to obtain an  $h$  with a high SNR one wants to maximize  $\text{SNR}(h)$ . This can only be done by varying the  $z_i$ 's and  $h'$ . A procedure to obtain a fairly good answer to this maximization question is to construct  $h$  from those parameters  $z_i$  and  $h'$  which solve the somewhat simpler minimax problem

$$\min_{z_i, h'} \max_{1 \leq k \leq m} \max_{x \in B_k} |\langle h_x, f_k \rangle|^2. \quad (7)$$

This is the author's own idea and own analysis of the problem. The mathematical and computational difficulty involved in the efficient solution of the digitized version of the minimax problem (7) cannot be overestimated. There is certainly no standard piece of software available which will find even local solutions of the minimax problem (7), let alone global solutions, even for fixed  $h'$ .

Consider the minimax problem (7) in which one fixes  $h'$ , say  $h' = 0$ . Just why would one expect a good answer to the minimax problem (7) merely by varying the  $n$  parameters  $z_1, \dots, z_n$ ? This can be seen by reformulating the question as one in planar geometry. Let  $(d_{ij})$  be the  $n$  by  $n$  matrix which is the inverse to the  $n$  by  $n$  matrix  $(\langle f_i, f_j \rangle)$ . Then for  $1 \leq i \leq n$ ,

$$a_i = d_{i1} z_1 \lambda_1 + \dots + d_{in} z_n \lambda_n \quad (8)$$

and

$$\begin{aligned} \langle h_x, f_k \rangle &= a_1 \langle (f_1)_x, f_k \rangle + \dots + a_n \langle (f_n)_x, f_k \rangle \\ &= (d_{11} z_1 \lambda_1 + \dots + d_{1n} z_n \lambda_n) \langle (f_1)_x, f_k \rangle + \dots \end{aligned} \quad (9)$$

$$\begin{aligned}
& + (d_{n1}z_1^{\lambda_1} + \dots + d_{nn}z_n^{\lambda_n}) \langle (f_n)_x, f_k \rangle \\
& = z_1^{\lambda_1} (d_{11} \langle (f_1)_x, f_k \rangle + \dots + d_{n1} \langle (f_n)_x, f_k \rangle) + \dots \\
& \quad + z_n^{\lambda_n} (d_{1n} \langle (f_1)_x, f_k \rangle + \dots + d_{nn} \langle (f_n)_x, f_k \rangle) \\
& = z_1 w_{x,k,1} + \dots + z_n w_{x,k,n},
\end{aligned}$$

where

$$w_{x,k,j} = \lambda_j (d_{1j} \langle (f_1)_x, f_k \rangle + \dots + d_{nj} \langle (f_n)_x, f_k \rangle). \quad (10)$$

The numbers  $w_{x,k,j}$  are complex numbers (in fact real numbers) and may be viewed as vectors in the plane. The numbers  $z_j$  are complex numbers of modulus 1 and so multiplication by them may be viewed as a rotation. Hence, these observations show that the solution of the minimax problem (7) may be reformulated as a special case of the following geometrical problem: given  $q$   $n$ -tuples of vectors in the plane,  $(v_{11}, \dots, v_{1n}), \dots, (v_{q1}, \dots, v_{qn})$ , find rotations  $R_1, \dots, R_n$  so that the maximal length of any of the  $q$  vectors  $R_1 v_{11} + \dots + R_n v_{1n}, \dots, R_1 v_{q1} + \dots + R_n v_{qn}$  is as small as possible. Note that without loss of generality  $R_1$  can be normalized to be the identity. Alternatively, one may view the minimax problem (7) as a problem about forces in the plane. Think of each  $v_{ji}$  as a force acting on a point particle at the origin. One then wants to choose the rotations  $R_1, \dots, R_n$  so that the biggest net force on the particle is as small as possible. For a hint as to how difficult this problem can be, merely concoct a few examples when  $n = 3$  and  $q = 3$ . In the numerical experiments described in the following section  $n = 36$  and  $q = 9,432,828$ .

Engineers and/or physicists perhaps will feel somewhat uncomfortable with reducing the design of a high SNR SDF down to the minimax problem (7). They perhaps will want to have a physical interpretation of the parameters involved

in its solution and will want to know if the SDF is correlating on some particular aspect of the training set images. These questions may have answers, but probably not. After all, it is an easy matter to give simple freshman calculus optimization questions for which the parameters involved in the solution have no apparent physical interpretation. There is even less reason to suppose that the solution to the enormously complicated minimax problem (7) is subject to a physical interpretation. Furthermore, asking such questions will be irrelevant if the filters given by the purely mathematical solution of the minimax problem (7) perform as desired.

### SECTION III

#### RESULTS OF NUMERICAL EXPERIMENTS

Numerical experiments on realistic data show that the solution of the minimax problem (7) can yield dramatic improvement in the SNR properties of SDF's. The images used in the following experiments were 36 256 by 256 real tank images. Even though the recipe given in the previous section does not depend on any particular property of the training set  $f_1, \dots, f_m$ , it may be of interest to describe in some detail the exact imagery used in these experiments. The 36 images originally were 512 by 512 images in a cluttered background. They were furnished on a computer magnetic tape, each image consisting of a string of  $512^2 = 262,144$  integers between 0 and 255 representing intensity levels at each pixel of the image. The tank images were then extracted from the background and placed into 256 by 256 arrays, using DeAnza array processing equipment. The original images had been edge-enhanced and then biased, so that all their entries were nonnegative, and then each pixel was discretized into one of 256 equal parts - hence the bit streams which appeared on the data tape. A somewhat inexact but hopefully fairly reasonable way to recover the original edge-enhanced tank images alone was then employed on the 256 by 256 images - for each image the average of the nonzero pixel values was computed and then subtracted from each of the nonzero pixels, leaving those pixels with 0 values unaltered. These edge-enhanced 256 by 256 tank images were then energy-normalized (i.e., they were divided by their length to make them into unit vectors.)

Use the notation of the previous section. In the following calculations,  $n = m$ , each  $B_i$  was taken to be an 11 by 11 box centered at the origin, each  $\lambda_i$  was chosen to be 1, and  $h'$  was chosen to be identically 0. The theory of the previous section is not affected by the somewhat ad hoc nature of the choice of  $h'$ ,  $\lambda_i$ , and  $B_i$ , and the programs used to solve the minimax problem (7) arrived at their answer only by varying the  $z_i$ 's.

The standard way of making an SDF is the process described in the previous section in which one takes every  $z_i$  to be 1 and  $h'$  to be identically 0. Such an SDF has terrible SNR properties against this training set. For example, the performance of this  $h$  against  $f_{24}$  gives an SNR less than 1.00, gives 1.02 against  $f_{19}$ , and against  $f_3, f_4, f_5, f_6, f_7, f_8$ , and  $f_{25}$  gives an SNR of less than 1.25. However, the author has used the recipe of the previous section to construct an SDF with an SNR of at least 7.37 against each  $f_i$ . The construction of this latter SDF was done by a sequence of rather elaborate computer programs which took a great deal of disk space and approximately 300 hours of CPU time to run on a VAX 11-780/VMS 4.2. As explained in the opening section, this long computation time is not really a cause for concern.

Note that 16.00 is certainly a very plausible upper bound for the SNR of any SDF filter  $h$  made from energy-normalized images and with each  $\lambda_i$  equal to 1, for the inner product of  $h$  with each training image quarter is a complex number whose expected modulus is approximately  $1/4$  and so whose expected modulus squared is approximately  $1/16$ . Practically, an SDF with an SNR of close to 8.00 might be attained.

The author recently has tested the performance of these new SDF's versus

the standard recipe in scenes with a cluttered background. One should view these experiments as trying to find a real tank in a junkyard of tank parts. In what follows the tank imagery used is the same as the tank imagery referenced above. Select one of the 36 256 by 256 tank images at random and place it in the middle of a 512 by 512 scene. This 256 by 256 tank image is surrounded by twelve 128 by 128 blocks. For each one of these blocks, select one of the 36 tank images at random, select one of the 128 by 128 quadrants of the latter image at random, and place it into the 128 by 128 block in the 512 by 512 image. Do this for all 12 128 by 128 blocks. One winds up with an approximation of a true tank located in the middle of a junkyard of randomly chosen tank quarters. Examples of such cluttered scenes are given as Random Scene 1 in Figure 1 and as Random Scene 2 in Figure 10. In these scenes only those pixels whose magnitude is at least 0.20 of the biggest pixel are illuminated. The author computed and displayed on a VHR19/6120 graphics terminal, in a variety of manners and aspects, the correlation plane outputs from an SDF filter made with the usual recipe, say SDF1, against a random sequence of junkyard scenes and compared them to the correlation plane outputs of an SDF filter made with the new recipe, say SDF2. Recall that these correlation plane calculations done via an FFT are actually done on a doughnut or the plane tiled with these 512 by 512 junkyard scenes, so there was a good possibility that the SDF's could get confused. This happened quite often with SDF1, but not once so far with SDF2. Approximately 200 independent runs of this experiment have been conducted and SDF1 gave its major correlation to clutter in the background approximately 25 percent of the time. SDF2 has never given a false major signal against any of these same junkyard scenes.

In fact, in no instance tested so far has SDF2 performed worse than SDF1 and in most instances has performed dramatically better. SDF2 so far has always picked the true tank from the junkyard and has given a very sharp recognition signal, much better than any signal coming from the surrounding clutter. This good fortune certainly cannot continue for all of the approximately  $36^{13} \times 4^{12}$  random junkyard scenes, but it does strongly suggest that on a statistical basis SDF2 performs much better than SDF1.

Figures 2, 3, 6, 7, 11, 12, 13, and 14 display some aspects of typical filter output correlation planes. Each output plane is a 512 by 512 array of nonnegative numbers representing the correlation intensities given by either SDF1 or SDF2 as they scan Random Scene 1 or Random Scene 2. Think of each output plane as representing a mountain range. Each of these figures is a graph which gives the outline one would see by viewing the mountain range in a south-to-north direction or a west-to-east direction. More precisely, each figure is 512 pixels wide, and the height of the graph above each pixel is the maximum intensity one would see along the column corresponding to the pixel if the view is south-to-north or is the maximum intensity one would see along the row corresponding to the pixel if the view is west-to-east. In all of these figures the signal coming from the tank in the cluttered scene is the central spike and has height 1. The largest height in each picture is displayed as 750 pixels high. Figures 2 and 3 show that SDF1 gives a false major signal to clutter in Random Scene 1, and Figures 6 and 7 show that SDF2 does not make such a mistake. Figures 11 and 12 show that SDF1 gives a very large false signal to the clutter in Random Scene 2 whereas Figures 13 and 14 show that SDF2 has no trouble distinguishing the true tank from clutter. Figures 2, 3,



11, and 12 are typical of what goes wrong with SDF1 in 25 percent of these tests. Figures 6, 7, 13, and 14 are characteristic of SDF2 in every instance so far tested.

Horner and Gianino, Reference 7, have recently suggested using as correlation filters the phase-only versions of SDF filters. These filters are manufactured by starting with an SDF filter  $h$ , taking its Fourier transform  $F(h)$ , saving only the phase of  $F(h)$ , say  $P(F(h))$ , and then taking as one's filter  $F^{-1}(P(F(h)))$ . They report considerable improvement in the output correlation plane with these phase-only filters versus that produced by the SDF filter alone. The author manufactured the phase-only versions of SDF1 and SDF2, say POSDF1 and POSDF2, and tested their performance against a sequence of randomly generated junkyard scenes. Unfortunately, both POSDF1 and POSDF2 performed very unsatisfactorily in most tests, giving, for instance, a sharp major correlation to empty portions of the scenes in several instances. In no instance tested has POSDF1 or POSDF2 performed as well in target recognition as SDF2. Figures 4, 5, 8, and 9 represent the output correlation planes of POSDF1 and POSDF2 against Random Scene 1 and are typical of their performance in just about every instance tested. In these figures the maximum intensity is displayed as 750 pixels high, and the intensity given by the filters against the true tank is not a set level, as opposed to the situation with SDF1 and SDF2, where the intensity given by the filters against the true tank is 1. There may well be overwhelming advantages to phase-only filters over the recipe described in this document. However, the author's conclusion from these tests is that the recipe described in this paragraph certainly is not the way to make them.

The author also tested the performance of SDF1 and SDF2 against a sharp false signal by examining the correlation plane outputs of the two filters against an input scene consisting of a spike of height 1 at the origin. In both instances the signal from the origin was practically buried in noise in the background.

These experiments give considerable hope that correlation filters manufactured with the recipe detailed in this document will give superior performance in practice.

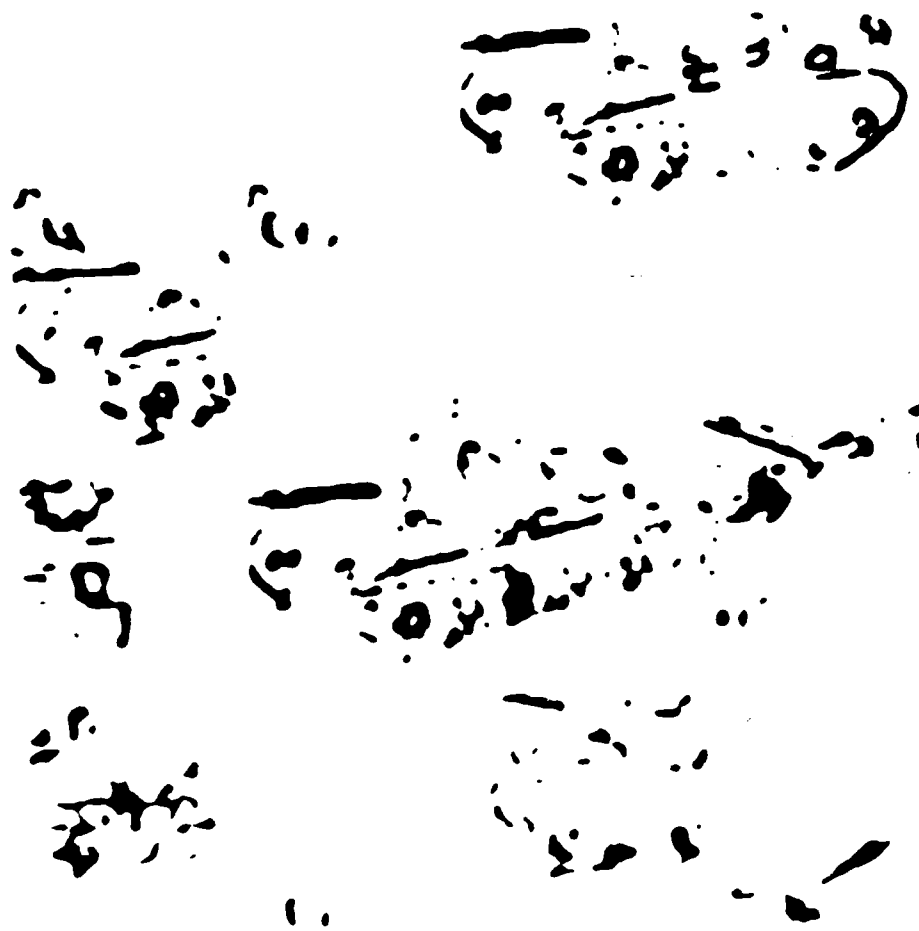


Figure 1. Random Scene 1



Figure 2. SDF1 vs Random Scene 1, S-N View

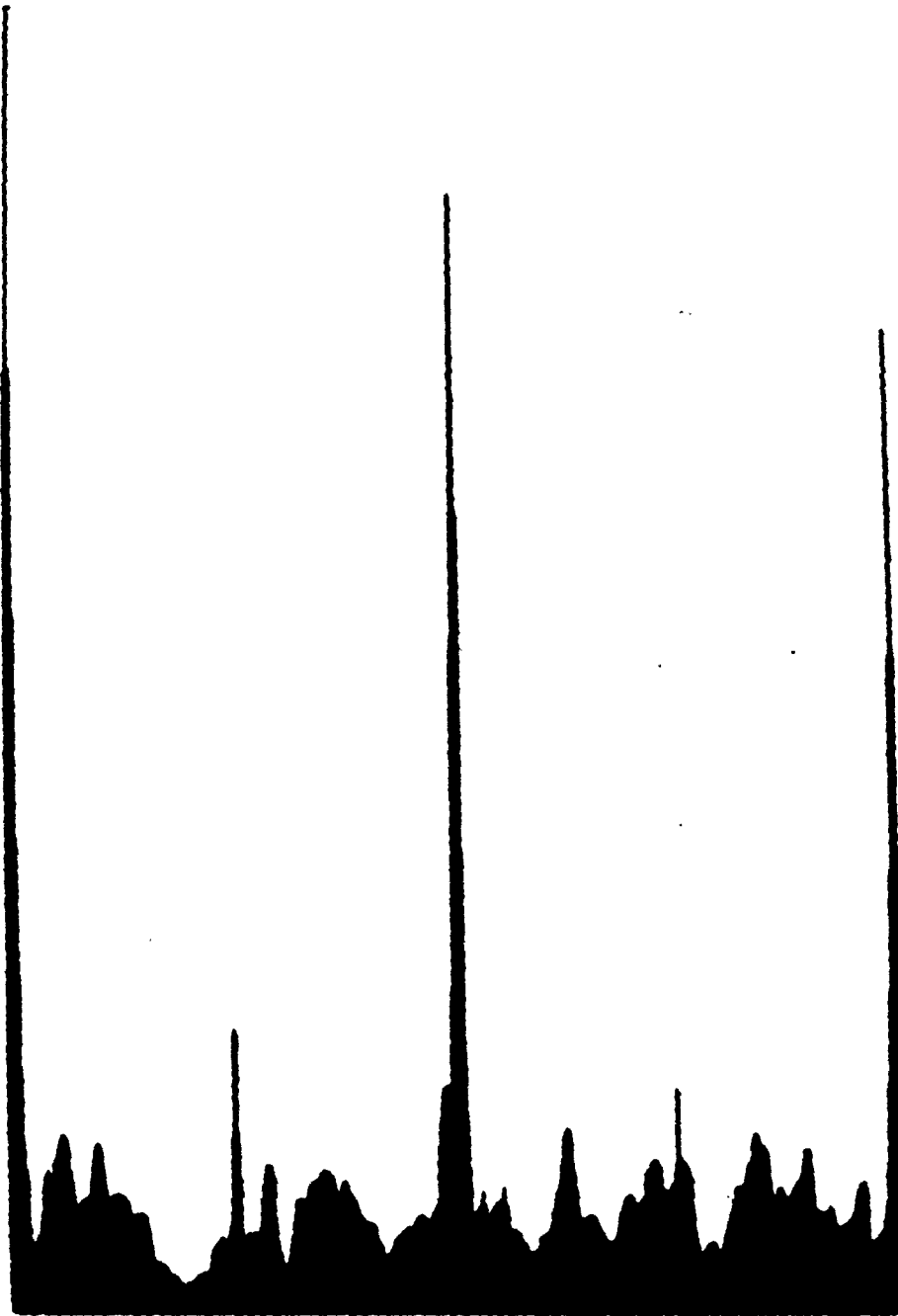


Figure 3. SDF1 vs Random Scene 1, W-E View

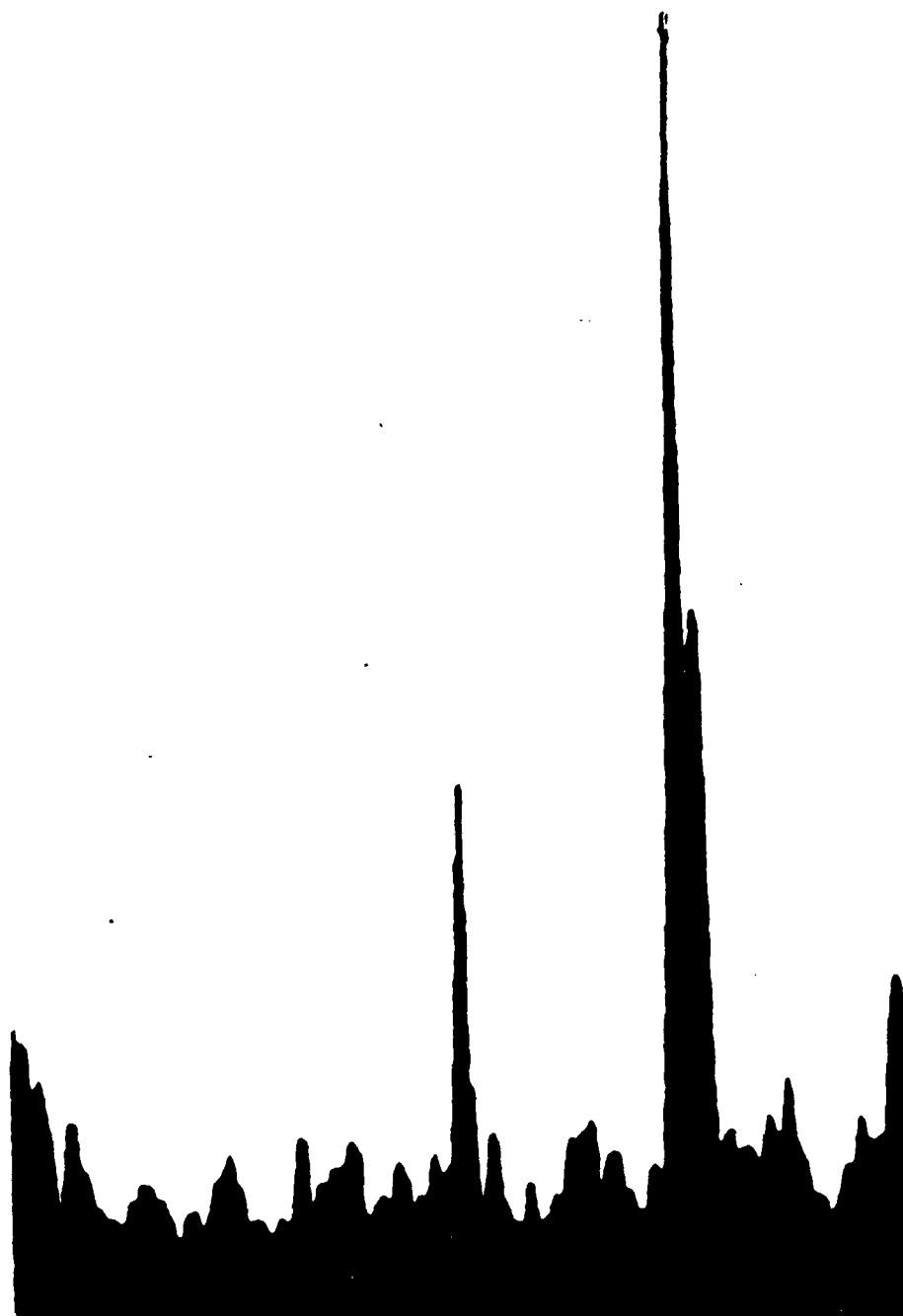


Figure 4. POSDF1 vs Random Scene 1, S-N View

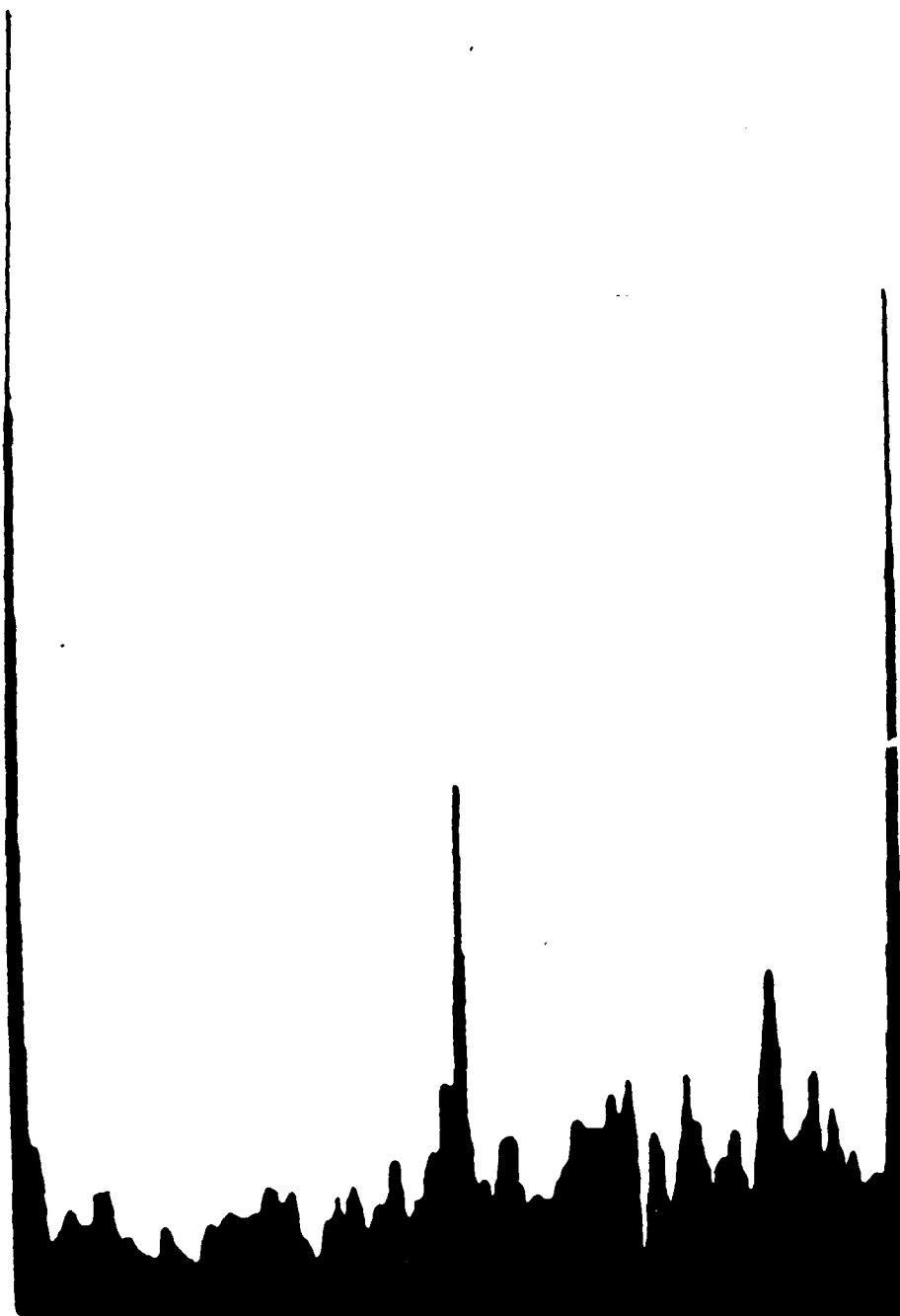


Figure 5. POSDF1 vs Random Scene 1, W-E View



Figure 6. SDF2 vs Random Scene 1, S-N View



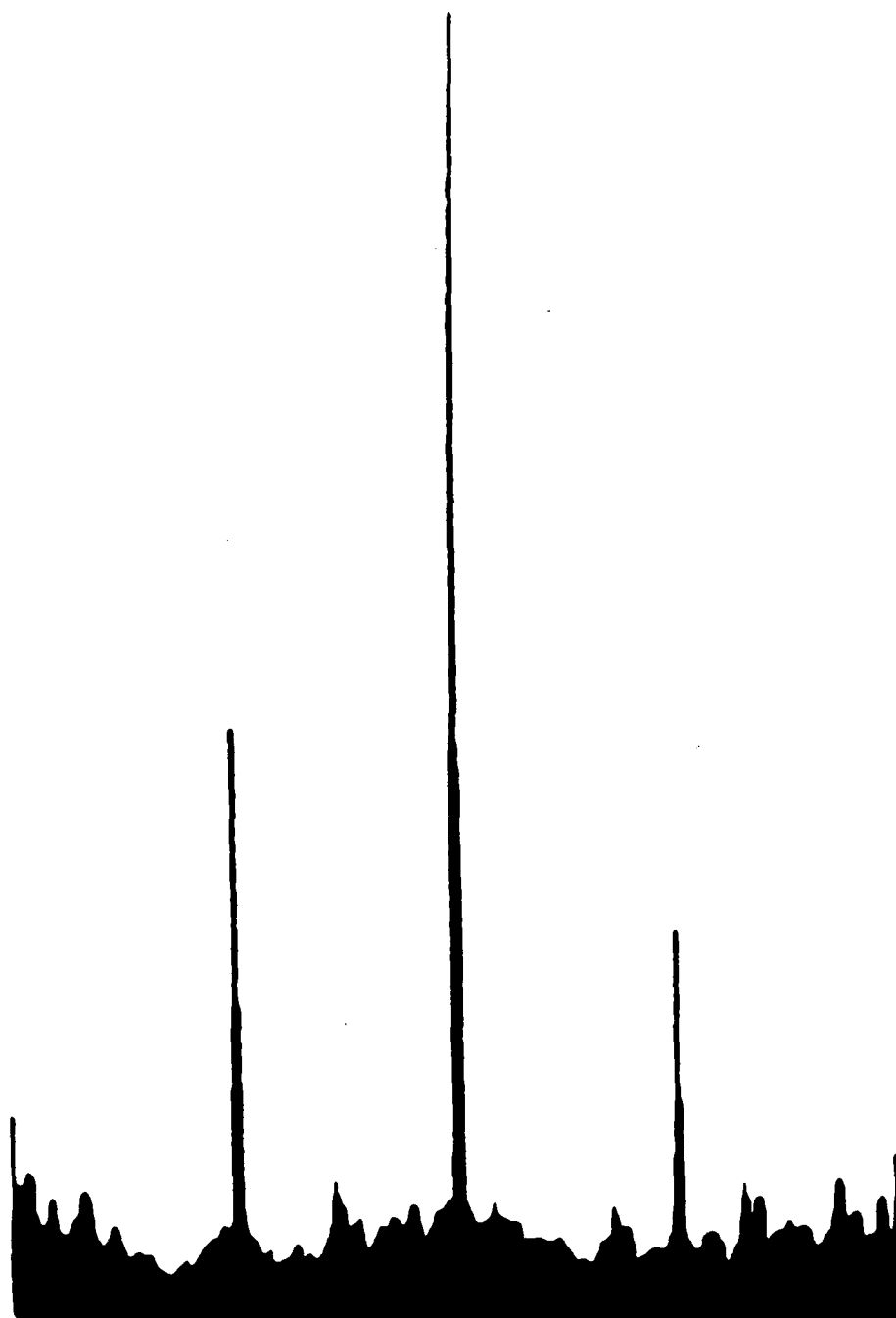


Figure 7. SDF2 vs Random Scene 1, W-E View

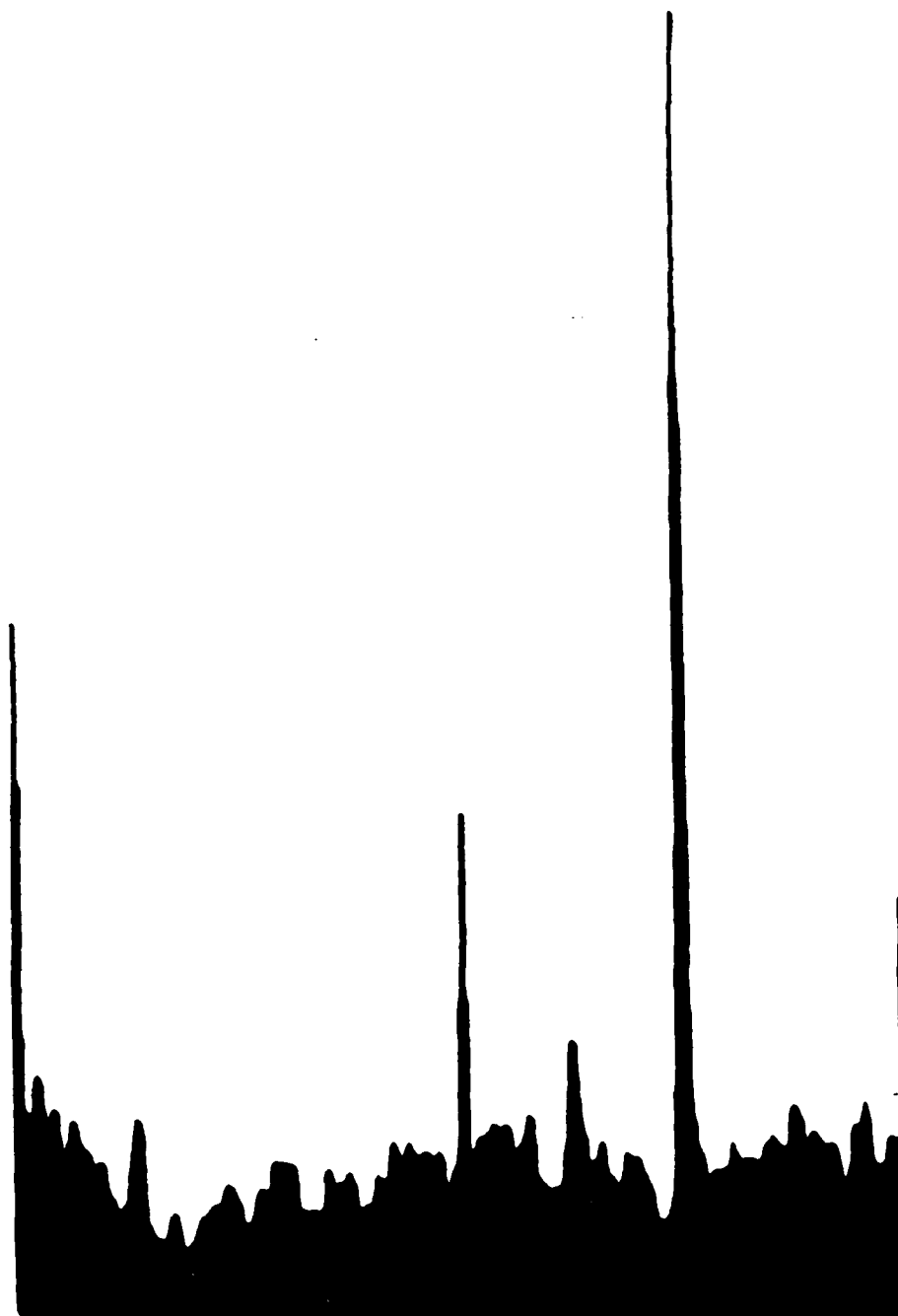


Figure 8. POSDF2 vs Random Scene 1, S-N View

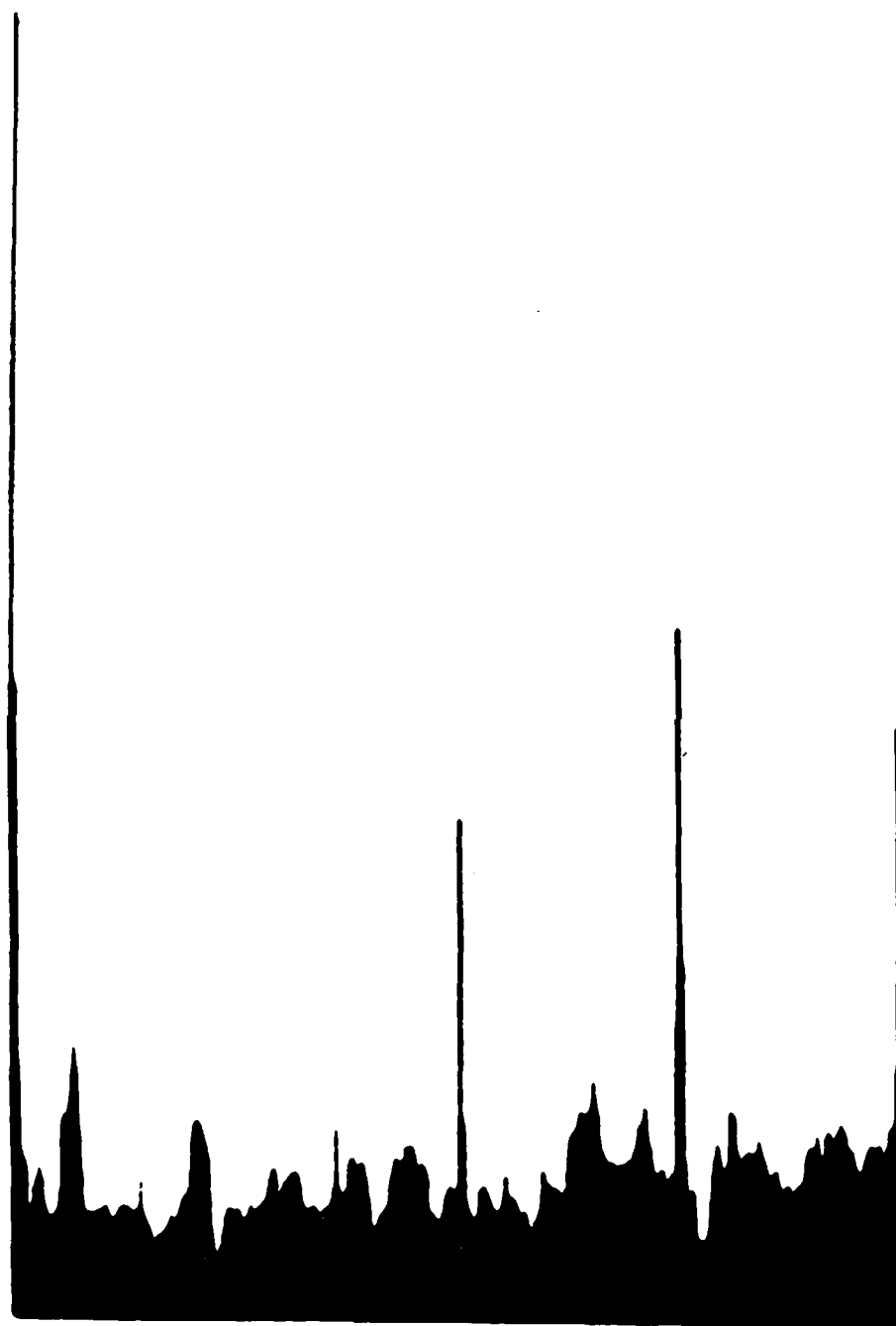


Figure 9. POSDF2 vs Random Scene 1, W-E View

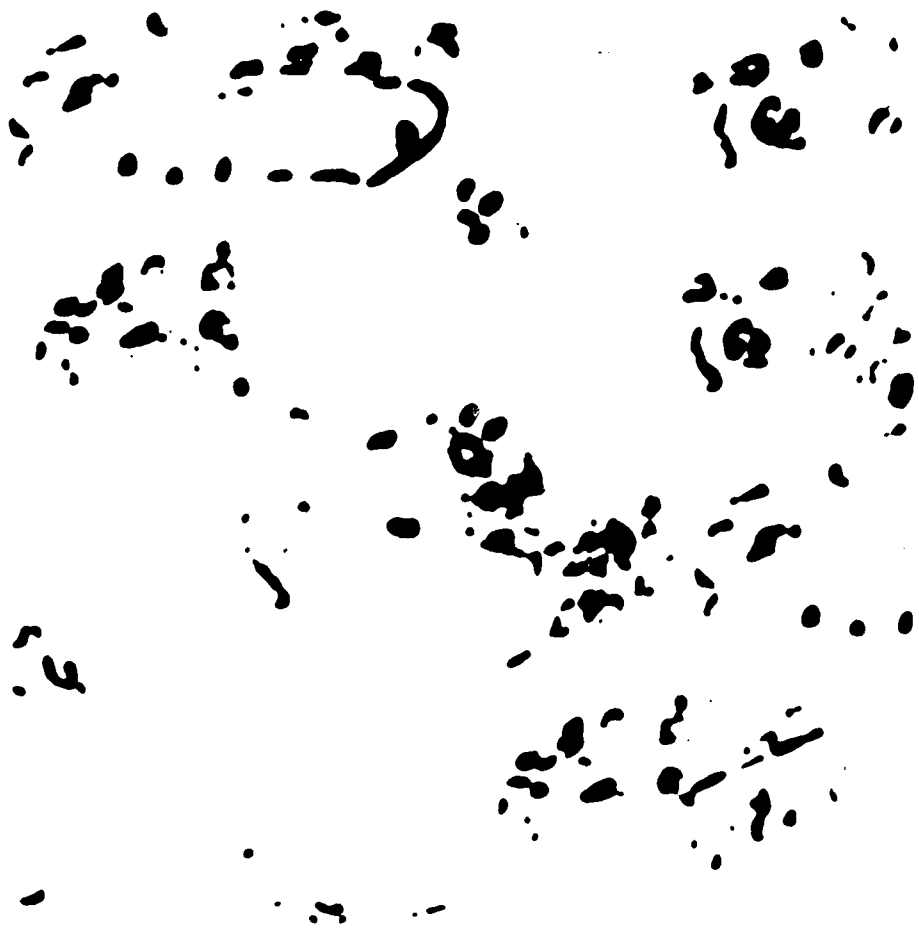


Figure 10. Random Scene 2

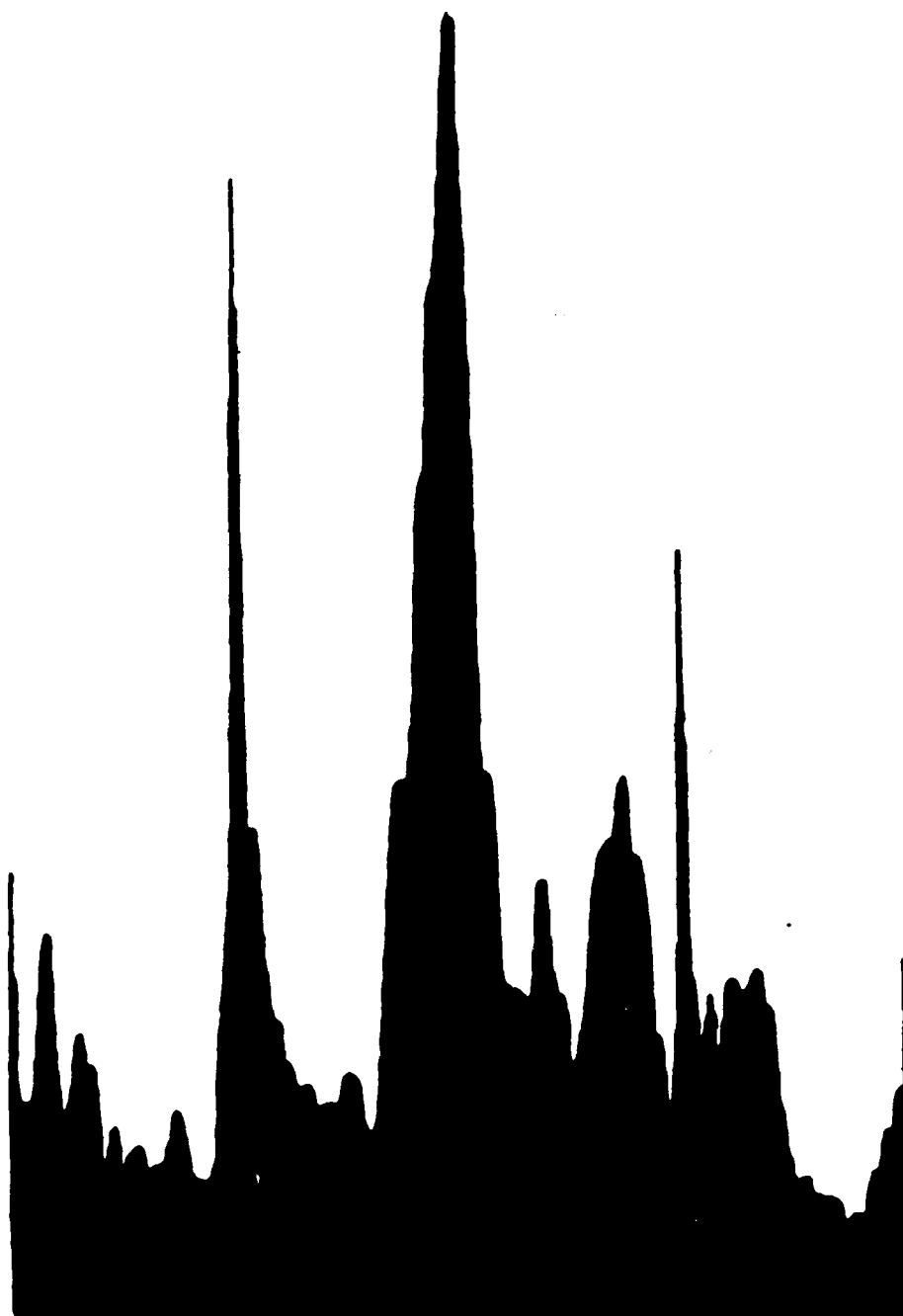


Figure 11. SDF1 vs Random Scene 2, S-N View

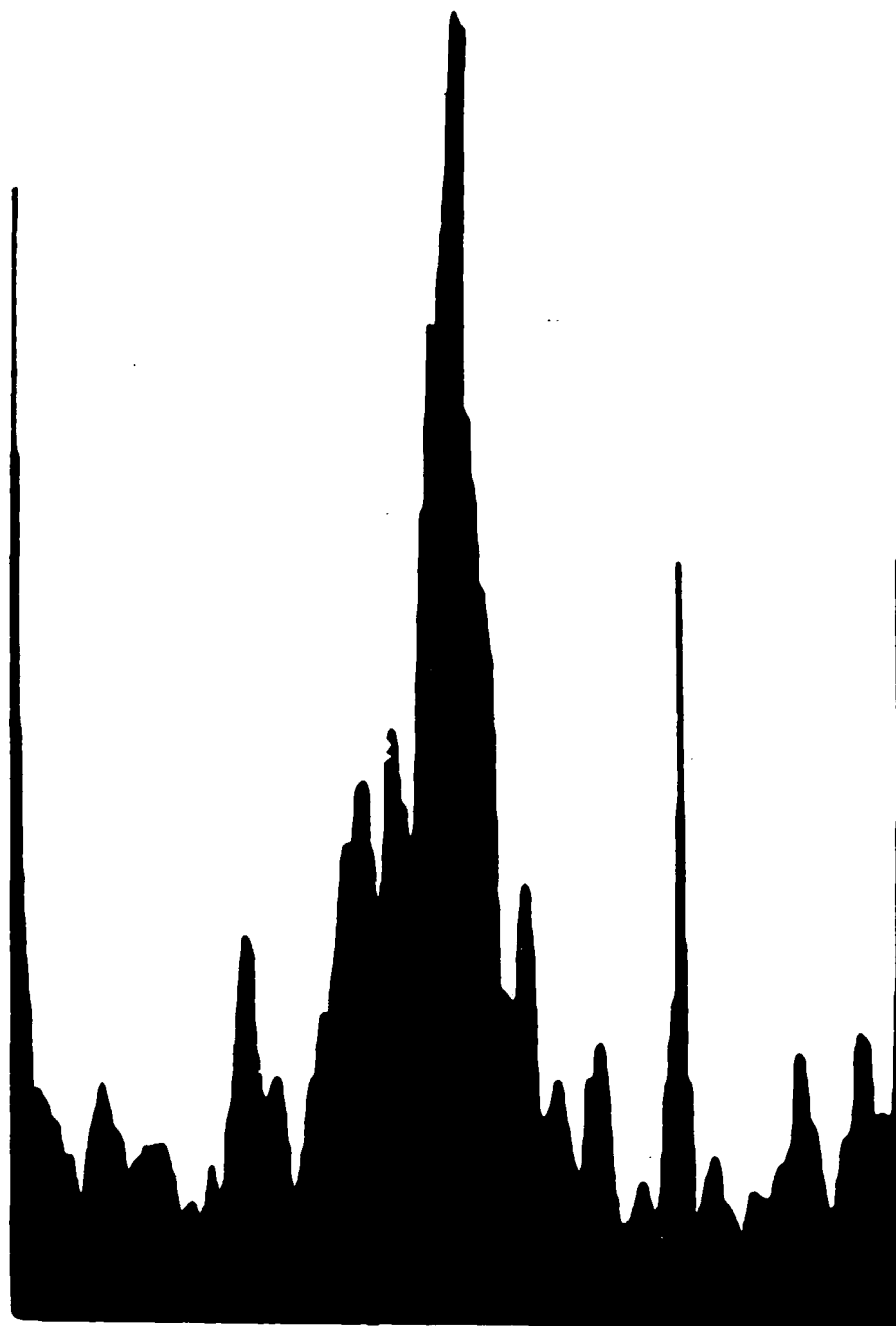


Figure 12. SDF1 vs Random Scene 2, W-E View



Figure 13. SDF2 vs Random Scene 2, S-N View

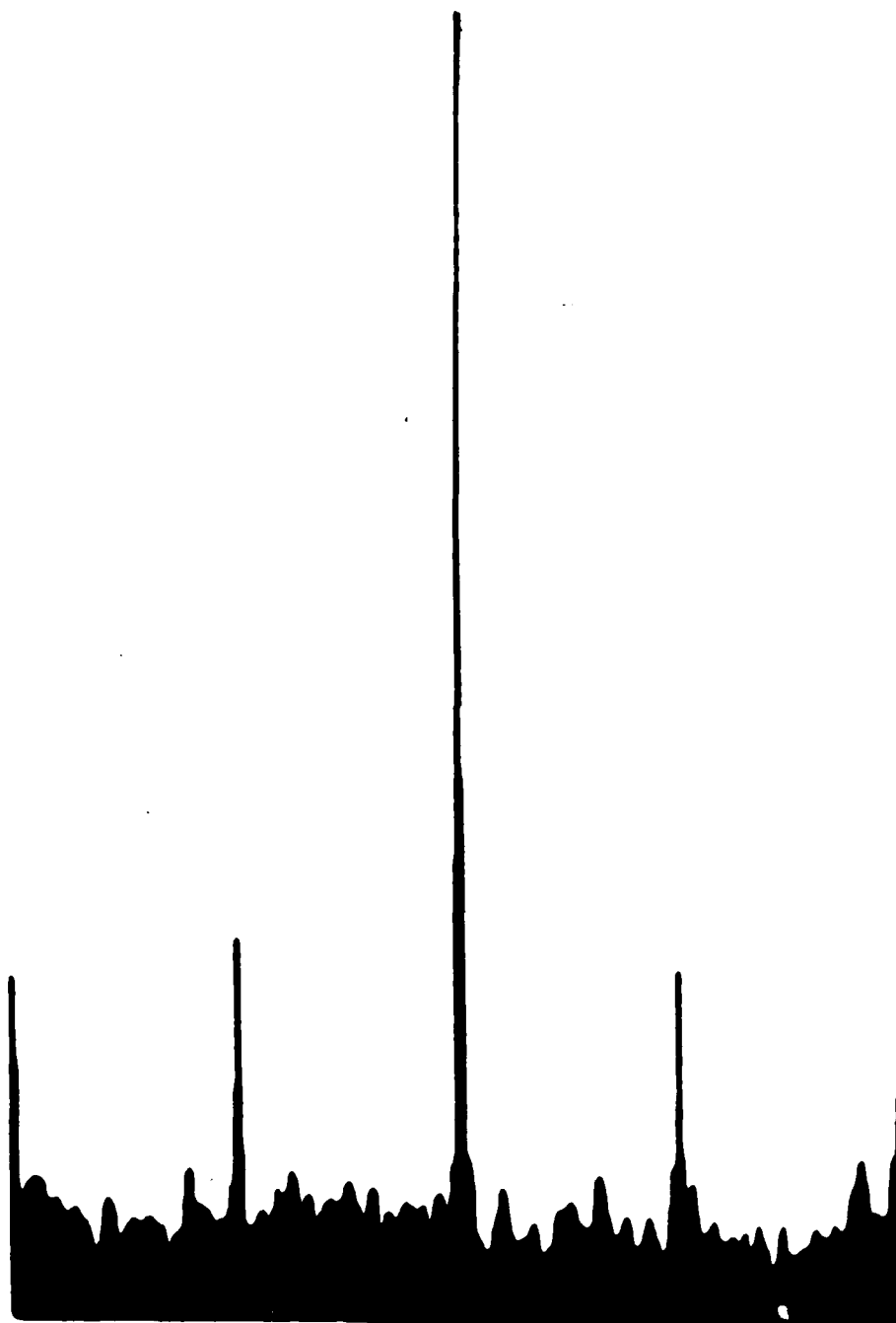


Figure 14. SDF2 vs Random Scene 2, W-E View



## SECTION IV

### OTHER ISSUES, WORK, AND APPROACHES

Some sort of edge-enhancement is necessary on a training set of images if one wants a high SNR correlation filter. The usual simple example given to justify this assertion is that a uniformly filled in square and a uniformly filled in circle of roughly the same size will correlate quite well with one another, even though they are quite different objects. This would not be true if they were reduced only to their edges. This example shows that some sort of edge-enhancement is necessary for the training image in matched filters, let alone for the training images in the much more complicated SDF filters.

Given that some edge-enhancement is necessary, then shouldn't more edge-enhancement be even better? If the tentative answer is yes, then one plausible way to obtain filters whose output correlation plane resembles a delta function centered over the target when the target is centered in the filter input plane is to make the filter from a training set of images which are extremely edge-enhanced. This train of thought leads to difficulties. Edge-enhancement operators should work roughly as follows. View an image as the graph of a function  $f$  on the plane. Let  $D_x$  and  $D_y$  be the standard partial derivative operators along the  $x$  and  $y$  axes. It is plausible that  $D_x(f)$  and  $D_y(f)$  should be small on the interior of uniform objects, but should be large when crossing the boundary of a uniform object. Hence, if  $P(D_x, D_y)$  is any constant coefficient differential operator without constant term on functions of two variables, then there is a good chance that  $P(D_x, D_y)(f)$  somewhat

resembles  $f$  with the interior of uniform objects eliminated but with edges enhanced. There is a strong plausibility argument that every true edge-enhancement operator must be of this form. Let  $E$  be any edge-enhancement operator. Then  $E$  should be a rule which assigns to an image  $f$  another image  $E(f)$  with the properties: (i)  $E$  is linear ( $E(af + bg) = aE(f) + bE(g)$  for all images  $f$  and  $g$  and constants  $a$  and  $b$ ); (ii)  $E$  is continuous (if  $f$  is close to  $g$ , then  $E(f)$  should be close to  $E(g)$ ); (iii)  $E$  annihilates the constants ( $E(C) = 0$  for any constant image  $C$ ); (iv)  $E$  is translation invariant; and most importantly, (v)  $E$  is local ( $E(f)(x)$  only depends on the values of  $f$  arbitrarily close to  $x$ ). If one admits these plausible assumptions for an edge-enhancement operator  $E$ , then it is an easy exercise in the Schwartz theory of distributions (Hoermander, Reference 8) that  $E$  must be a constant coefficient differential operator without constant term. Such an edge-enhancement operation might be implemented optically, for it is equivalent to multiplication by  $P(-ik_1, -ik_2)$  in Fourier transform space. Notice that after any operation of this sort, the Fourier transform of the processed image vanishes at the origin. For example, if  $P$  is the Laplacian  $D_x^2 + D_y^2$ , then  $P$  is equivalent to multiplication by  $-|k|^2$  in Fourier transform space. Hence, there are limits to how much edge-enhancement one can do without greatly distorting the original image, for all edge-enhancement operators involve eliminating or suppressing low spatial frequencies, thus emphasizing the high spatial frequencies and putting a great deal of weight on the noise in the scene. This suggests that even a matched filter would not perform very well if it were made from a training image which was extremely edge-enhanced. Perhaps this difficulty can be removed by truncating

$P(-ik_1, -ik_2)$  at a certain level outside of some disk about the origin. An extreme case of this is multiplication by  $1 - \delta_{(0,0)}$  in Fourier transform space, which will be referred to as minimal edge-enhancement. The problem with this truncation operation is that multiplication by such a truncated polynomial in Fourier transform space would no longer be a local operation when transferred to physical space - i.e., its effect on a given pixel would depend on other pixels quite far away and not just on arbitrarily close ones. This seems to contradict one's intuition as to what properties an edge-enhancement operator usually should have.

It is quite possible that the recipe for SDF's given in this document will make moot all questions as to what kind and how much edge-enhancement should be done on a training set of images. Numerical experiments conducted on filters made with the recipe outlined in this document give filters with a quite high SNR on training sets with only minimal edge-enhancement - only a small disk about the origin in Fourier transform space need be set to 0, perhaps even just the origin itself, in order to obtain filters with a high SNR. Numerical experiments on this important question are continuing. If they are as successful as early experiments indicate, then the recipe given in this document will automatically eliminate the questions as to what kind of edge-enhancement and how much edge-enhancement is enough on a training set in order to get a low noise correlation filter. The answer will be only an absolutely minimal amount. After all, the major reason for doing any edge-enhancement at all is to construct high SNR correlation filters.

There is another important technical advantage to using images in the training set which are at least minimally edge-enhanced. If each image  $f_1$  in

the training set is edge-enhanced, then at the very least this implies that the Fourier transform  $F(f_i)$  vanishes at the origin, or equivalently that the average pixel value  $av(f_i)$  is 0. Since  $h$  is a linear combination of the  $f_i$ 's, the average pixel value  $av(h)$  is also 0. Now let  $g$  be any constant image - i.e., every pixel value of  $g$  is some fixed constant  $C$ . But then  $\langle h, g \rangle = Cav(h) = 0$ , so  $h$  will give zero correlation to any constant image. Note that if  $av(h)$  were not 0, then  $Cav(h)$  could be quite large and  $h$  would give major correlations with obviously false targets. It is for this reason that at least a minimal amount of edge-enhancement be done to the images in the training set for  $h$ . On the other hand, if  $av(h) = 0$ , then for any image  $g$  which is the same size as  $h$ ,  $\langle h, g \rangle = \langle h, (g - av(g)) \rangle$ . Note that the image  $g - av(g)$  is a minimally edge-enhanced version of the original image  $g$  since  $av(g - av(g)) = 0$ . This is important in practice, for suppose the SDF  $h$  is made from a training set of images which are 32 by 32 in size and minimally edge-enhanced. If  $h$  is searching for targets in a 512 by 512 scene  $g'$ , then even if  $g'$  is extremely edge-enhanced, its resulting generic 32 by 32 subscene certainly need not have even its average pixel value 0. But the previous analysis shows that if  $h$  is made from a training set of weakly edge-enhanced images, then no edge-enhancement at all need be done on  $g'$  for the filter  $h$  to behave as if each 32 by 32 subscene was weakly edge-enhanced.

Many practitioners of this subject insist that the images used in the training set of an SDF be energy-normalized. Energy normalization is equivalent to replacing each image  $f_i$  by  $f_i / ||f_i||$ , so that the total energy in each training image is 1. (One should first edge-enhance and then energy-normalize the image. The two operations should be done in this

particular order, for they do not commute.) The usual reason given for doing this is to reduce the variation in the climatic effects in which the targets are located. This is plausible since Stefan's Law implies that the total amount of radiation emitted by a black body is proportional to  $T^4$ , most of which is in fairly low (infrared) frequencies for normal temperatures. Thus small changes in the temperature of a black body result in rather high changes in the amount of infrared radiation it emits. Furthermore, since one a priori has no reason to know which aspect of a target one will encounter, it is plausible that each image used in the training set should be normalized to be of the same energy to initially give each training image equal weighting in the design of the filter.

The initial conditions (1) were introduced so that the SDF filter would work if employed in a threshold detector. In order for such a threshold device to work well it probably must be necessary to optically energy-normalize the input scenes to the filter. There are at least two reasons why this practice should be done with a certain amount of caution. Since the SDF will be made from energy-normalized images, say 32 by 32 pixels in size, it will be looking for energy-normalized images, 32 by 32 pixels in size. But an energy-normalized 512 by 512 image certainly does not in general have each 32 by 32 subscene proportionally energy-normalized. If the object sought is the brightest object in the 512 by 512 scene, then energy-normalizing the entire image will leave the target subscene more than proportionally energy-normalized, and no harm will result if one is merely searching for the largest correlation peak. But if there is a much brighter object, say a fire, in the upper left hand corner of the image, and the target

is in the lower right hand corner, then energy-normalizing the entire scene perhaps will make the image of the object sought so faint as to be useless. The probability of this unfortunate effect will surely be reduced if one uses correlation filters with the highest possible SNR, for the possible higher correlation given by an intense false target will be partially or totally offset by the higher SNR of the filter. Even extreme cases of this phenomena can be eliminated if it is feasible to optically energy-normalize those subregions of the input plane which are the size of the training images used to construct  $h$ .

It must be noted that there is a difficulty, though a low probability one, with this SDF correlation method. The Schwarz inequality implies that the SDF will have largest correlation with unit vectors that do not look like the targets in the training set, but instead look like the SDF itself. The difference might be quite pronounced since, to a crude first approximation, the length of  $h$  is approximately  $m^{1/2}$  if there are  $m$  edge-enhanced and energy-normalized images in the training set for  $h$ . For the SDF filter process to really work with a low probability of error, one must make an act of faith that there are very few real objects which look more like the SDF than the target images themselves.

There are many papers in the literature on the manufacture of SDF's whose philosophy is as follows. Given a training set of images, try and find (usually by some sort of orthogonalization process) those images in the collection which are the important ones and make an SDF from these. The proponents of this method find this philosophy to be plausible by claiming that the eigenvalue list of the matrix  $(\langle f_i, f_j \rangle)$  contains one or two of

comparable size and that all others are incredibly small in comparison. It must be emphasized that this is true only if the  $f_i$ 's are not edge-enhanced. Here is a very crude engineering argument for this fact. Suppose that the  $f_i$ 's are energy-normalized. Since they are not edge-enhanced, then the images  $f_i$  are solid blobs of unit energy. Since this is the case,  $\langle f_i, f_j \rangle \approx 1$  to a first approximation, and so  $(\langle f_i, f_j \rangle) \approx$  an  $n$  by  $n$  matrix of 1's, to a first approximation. It is a simple matter to check that the eigenvalue list of the latter matrix is  $n, 0, 0, \dots, 0$ . Recall that it has been emphasized earlier in this section that one must use a training set of images which are at least weakly edge-enhanced.

Now suppose that the images  $f_i$  are edge-enhanced and energy-normalized and let us again employ a crude engineering argument to determine a first approximation to the eigenvalue list of  $(\langle f_i, f_j \rangle)$ . Since the  $f_i$ 's are edge-enhanced and energy-normalized, they resemble bizarrely twisted coathangers suspended in space with unit energy. Hence,  $\langle f_i, f_i \rangle = 1$  and  $\langle f_i, f_j \rangle \approx 0$ , to a first approximation, if  $i \neq j$ . Hence, to a first approximation, in this case  $(\langle f_i, f_j \rangle) \approx$  the identity matrix, and the eigenvalue list for the latter matrix is  $1, 1, \dots, 1$ . In any event this argument suggests that all of the eigenvalues for  $(\langle f_i, f_j \rangle)$  have approximately the same order of magnitude, and that most if not all of the images  $f_i$  are of equal importance. This is confirmed in actual practice by at least one example - for the 36 edge-enhanced and energy-normalized tank images used in the numerical experiments of the previous section, the eigenvalues of the 36 by 36 matrix  $(\langle f_i, f_j \rangle)$  range between 0.6 and 2.2. Thus, attempts to make a good SDF by choosing the 6 most important out the 36 edge-enhanced and

energy-normalized tank images are definitely doomed to failure. After all, there is something suspicious about a philosophy which asserts that one can do a better job at recognizing images by throwing away information about them. These qualitative statements were verified in an objective manner through very extensive numerical experiments carried out by the author during the summer of 1984.



## SECTION V

### CONCLUSIONS AND RECOMMENDATIONS

Extensive numerical experiments strongly suggest that applying the recipe given in this document to a training set of weakly edge-enhanced and then energy-normalized images yields an SDF filter which gives very sharp recognition signals to the training set images and has a very low probability of committing an error. Such filters might well prove to be useful if they can be implemented in a high speed reusable spatial light modulator, thereby storing information about  $10^3$  to  $10^4$  training set images, particularly if input imagery to the filter can always be globally and sometimes locally energy-normalized.

The numerical experiments detailed in this report were only carried out on one set of tank imagery. These experiments should be repeated for many other sets of imagery to see if high SNR SDF filters can be made from them by the recipe given here. Furthermore, the successful implementation of these filters in a real optical system can only be done via computer-generated holograms. Whether or not this can indeed be done should be intensively studied.

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